Belief Preference in Graph Model for Conflict Resolution with Two Decision-Makers

Hanlin You, Mengjun Li, Jiang Jiang, Bingfeng Ge, Xueting Zhang, and Jianguo Xu
College of Information System and Management, National University of Defense Technology, Changsha, China
hlyou1990@hotmail.com, mjli11260744@gmail.com, jiangjiangnudt@hotmail.com
bingfengge@nudt.edu.cn, zhangxueting0807@163.com, jgxu1992@hotmail.com

***Abstract*—In order to deal with the subjectivity, epistemic uncertainty and incompleteness of input data, a novel methodology is proposed for interactive decision making between two players based on the graph model for conflict resolution (GMCR) with belief preference. First, a new GMCR model is calibrated, in which belief preferences are generated via evaluating the utilities of feasible states. Second, new definitions of unilateral improvements and four stability concepts are presented for conflict analysis. Finally, an interactive decision case on a weapon system of systems (WSoS) architecting is studied to demonstrate the feasibility of the proposed methodology.**

***Index Terms*—Belief preference, graph model for conflict resolution (GMCR), interactive decision making, weapon system of systems (WSoS).**

I. Introduction

Conflicts are ubiquitous in various management issues, such as system architecture design, resource allocation and negotiation. For different decision-makers (DMs) in a specific situation, the divergence in their preferences of alternatives is inevitability due to their own position and benefits [1]. The stakeholders with different knowledge and experience are, in essence, interrelated and interactive. Consequently, the key to improve holistic performance is to eliminate conflicts for suitable compromises and informed consensus through an interactive decision procedure including both competition and cooperation [2].

A proper approach for conflict analysis strives to capture and describe key characteristics of interactive decision cases. Comparing with DMs and their feasible options, the preference information is more difficult to obtain and measure. The graph model for conflict resolution (GMCR) provides a flexible and easy-to-use tool to handle this problem [3]. One can model and analyze a conflict with minimal preference information requirements (e.g., subjective, qualitative). Accordingly, it is widely utilized widely in broad and pervasive system domains [4].

However, due to the shortage of support information and limited rationality of DMs, it is improper to assume that the subjective preference judgments should be always deterministic [5]. Therefore, there is an important requirement for conflict modeling and analysis to identify and describe uncertainties, both aleatory and epistemic, in decision issues [6]. The former one has been discussed in recent literature [7], while the latter still needs to investigate. The reminder of this paper is organized as follows. Related literatures are reviewed in Section II and the conflict model with belief preference is introduced in Section III. Then, methodologies for conflict analysis with two DMs are proposed in Section IV and an illustrative example is utilized to demonstrate the foregoing approach in Section V. Finally, appropriate conclusions are drawn in Section VI.

II. Literature Review

Due to the increasing complexity, it is difficult to conduct capability-based system-of-systems architecting which involves different requirement criteria and stakeholders. Not only the assessment indicators for alternatives, which origin from the preference of DMs, are multidimensional and conflicting, but also the decision support intelligence include both certain and uncertain information. A multi-view approach was researched to describe the requirement of a weapon system of systems (WSoS) and capture knowledge and experience from different stakeholders [8]. In order to deal with different kinds of uncertainties, such as ignorance and subjectivity, the evidential reasoning approach was utilized to evaluate capabilities of a WSoS [9]. Moreover, Ge *et al.* [2] proposed a novel interactive portfolio decision analysis methodology for capability-based WSoS architecting using the GMCR.

As the theoretical basis of the interactive decision methodology, the complete formulation and related definitions for GMCR are first presented by Fang *et al.* [10]. Due to the requirement to analysis practical conflicts, the uncertainties in preference have been investigated. Li *et al.* extended the preference structure from a binary set to a triplet by introducing the uncertain relation [11] and presented an integrated algorithm for status quo analysis with uncertain preferences [12]. Furthermore, a new model was calibrated with hybrid preference structure combining strength and uncertainty and utilized in an environmental conflict [13]. For the quantitative description of uncertainties, the conflict models with preferences represented by fuzzy set, grey number and probability value were formulated and applied in different decision cases [7] [14] [15]. However, although these foregoing methodologies work well to tackle with the aleatory uncertainties, it is still inadequate for them to illustrate and reduce the epistemic uncertainties using support information and expertise.

The Dempster-Shafer (D-S) theory of evidence was established to reduce the epistemic uncertainties due to the deficient of support information and knowledge [16]. The related methodology excels at dealing with incomplete uncertain input data. Moreover, the evidential reasoning approach, investigated based on the D-S theory, utilized the belief structure to represent the uncertainties in subjective assessments [5]. Yang *et al.* proposed an aggregation algorithm to integrate the assessment results of basic decision attributions in the belief structure using the evidential reasoning approach [17] and improved this algorithm to satisfy four synthesis axioms [18]. Further, the belief rule-based inference methodology using the evidential reasoning (RIMER) approach was put forward to integrate the objective support information and subjective expertise [19]. With the if-then rule structure, input data is transformed into a belief distribution, which is capable to capture the uncertainties, incompleteness and vagueness. Chang *et al.* [20] proposed a structure learning approach for the belief rule base to improve RIMER and utilized it to evaluate the overall capability of a WSoS.

Although the two kinds of methodologies are applied successfully in respective area, it is difficult for current GMCR to formulate preferences with epistemic uncertainties and incomplete information, and for RIMER to establish an interactive framework to resolve conflicts of different DMs. Accordingly, a new approach, combining the advantages of GMCR and RIMER, need to be researched for interactive decision making.

III. CONFLICT MODEL WITH BELIEF PREFERENCE

There are three interrelate components in formulating a conflict model with belief preference, including the framework of GMCR, utility assessment for feasible states and belief preference generation. Approaches for these procedures are demonstrated as follows, respectively.

A. *GMCR Framework*

GMCR is an effective tool to depict the interactive decision procedure of multiple DMs. In the graph model, vertices represent the feasible states, which consist of the options of all DMs, while the arcs indicate the reasonable state transitions. Every DM can execute an available move in one step through his controlled arcs to maximize his interests. As a result, there are four key elements in the framework of GMCR, including the DMs, game states, state transitions and relative preferences [10].

Definition 1: A GMCR is represented as a four-tuple structure $G=(N,S,(A_i)_{i\in N},(P_i)_{i\in N})$, where

(1) $N=\{1,2,...,n\}, n\in[2,+\infty)$ is the set of DMs;

(2) S is the set of feasible states, which stems from the combinations of all options ($S\in[2,+\infty)$);

(3) for each $i\in N$, A_i indicates the set of reasonable state transitions which are controlled by DM i; and

(4) for each $i\in N$, P_i is the relative preference set for feasible sates of DM i.

In the traditional GMCR methodology, the preference relations are represented using a binary set $\{\succ_i,\sim_i\}$, where for $s,t\in S$, $s\succ_i t$ indicates state s is better than t for DM i, while $s\sim_i t$ means the DM is indifferent between the two states [10]. Based on this preference structure, the reachable lists of unilateral moves (UMs) and improvements (UIs) for DMs are defined as following:

Definition 2: For $i\in N$ and $s\in S$, DM i's reachable list of UMs from state s is represented as $R_i(s)=\{t\in S\,|\,(s,t)\in A_i\}$ and the reachable list of UIs is represent as $R_i^+(s)=\{t\in S\,|\,(s,t)\in A_i\wedge t\succ_i s\}$.

On these theoretical bases, a belief preference structure is proposed to improve the GMCR methodology in this paper. An approach based on RIMER is utilized to assess the utilities for feasible states and a novel assembling method for uncertain and incomplete data is presented to generate the relative preference information.

B. *Utility Assessment for Feasible States*

As mentioned above, the feasible state set is the selected subset of option combinations of all DMs. Accordingly, the option choices are considered as the input data for state assessment. The "preference tree" method to prioritize options and states is utilized in the conflict analysis software GMCR II successfully [22]. However, it is still inadequate to deal with quantitative but uncertain and incomplete prioritizing data. Therefore, the "if-then" rules are used to depict the preference relations between option choices and states, and an evidence-integrating approach based on RIMER is proposed to assess utilities for feasible states [17-19].

Definition 3: The option set is represented as O, where $O=\{o_1,o_2,...,o_{|O|}\},|O|\in[2,+\infty)$. Then, the consequent of the i^{th} state is indicated as $C_i=\{C_1,C_2,...C_{|O|})$. Assume that the universal set of decision criteria is $A=\{A_1,A_2,...A_n),n\in[2,+\infty)$, the evaluation value of decision criteria for the i^{th} state is demonstrated as $B_i=\{B_1,B_2,...B_n)$.

Definition 4: For $\forall i\in[1,n]$, the b^{th} "if-then" rules are written as

$$R_b:\ if\ B_1^b\wedge B_2^b\wedge...\wedge B_i^b,\ then\ D_b \quad or\ if\ B_1^b\vee B_2^b\vee...\vee B_i^b,\ then\ D_b \qquad (1)$$

where the D_b is the consequent in the b^{th} rule, which measures the utilities of feasible states. Moreover, D_b is represented as $\{(H_k,\beta_k),k=1,2,...,K\}$, and

(1) H_k is the k^{th} evaluation grade in a descending rank in which H_k is preferred to H_{k+1}; and

(2) β_k is the belief degree for the outcome evaluation grade H_k and $\beta_k\in[0,1],\sum_{k=1}^{K}\beta_k\in[0,1]$.

Nevertheless, it is infrequent that input state data satisfy "if-then" rules perfectly due to the limitation of the constructed rule base. For that reason, the input data are transformed to a belief structure and an equation is designed to calculate the belonging degree for each antecedent [19].

Definition 5: For $s\in S$, the input data of state s is defined as $X^s=(x_1^s,\varepsilon_1^s)\wedge(x_2^s,\varepsilon_2^s)\wedge...\wedge(x_n^s,\varepsilon_n^s)$, where the x_i^s is the evaluation value of i^{th} decision criterion and ε_1^s indicates its belief degree.

Definition 6: The α_{ij} indicates the belonging degree of input data x_i^s to B_{ij}, where B_{ij} is the j^{th} possible value of C_i according to the rule base and

$$T(x_i^s,\varepsilon_i^s)=\{(B_{ij},\alpha_{ij}); i=1,2,...,n,\ j=1,2,...,|B_{ij}|\}$$
$$\alpha_{ij}=\varphi(x_i^s,B_{ij})\varepsilon_i^s \Big/ \sum_{j=1}^{|B_i|}\varphi(x_i^s,B_{ij}) \quad (2)$$

The function $\varphi(x_i^s,B_{ij})$ is utilized to depict the similarity degree between x_i^s and B_{ij} and its equation is determined by the type and monotonicity of x_i^s. More details can be found in related literatures [17] [21]. Next, an approach to compute the weights of activated rules is demonstrated as follows [19] [21].

Definition 7: For the transformed results of input data to b^{th} rule $\{(B_i^b,\alpha_i^b); i=1,2,...,|B^b|\}$, let α^b as the matching degree to this rule, then

(1) if the antecedents are connected using the $\wedge$:

$$\alpha^b=\prod_{i=1}^{|B^b|}(\alpha_i^b)^{\bar{\delta}_{bi}},\ \alpha^b\in[0,\underset{i=1}{\overset{|B^b|}{Min}}\{\alpha_i^b\}]$$
$$\bar{\delta}_{bi}=\delta_{bi}\Big/\underset{i=1}{\overset{|B^b|}{Max}}\{\delta_{bi}\},\ \bar{\delta}_{bi}\in[0,1] \quad (3)$$

(2) if the antecedents are connected using the $\vee$:

$$h_i^b=\bar{\delta}_{bi}\,\alpha_i^b,\ \alpha^b(1)=h_1^b$$
$$\alpha^b(i+1)=\alpha^b(i)+(1-\alpha^b(i))h_{i+1}^b,\ \alpha^b=\alpha^b(|B^b|) \quad (4)$$

where the δ_{bi} indicates the importance of the i^{th} antecedent in the b^{th} rule.

Definition 8: Assume that there are M activated rules and the relative importance of the b^{th} activated rule is represent as θ_b. Then, the weight of this rule is defined as

$$w_b=\theta_b\alpha^b\Big/\sum_{i=1}^{M}\theta_i\alpha^i,\ w_b\in[0,1]. \quad (5)$$

In certain circumstances, the belief degree of the consequent of a rule needs to modify due to the incomplete of input antecedent belief distributions [21].

Definition 9: Assume that μ_b is the modify factor of the b^{th} rule, $\bar{\beta}_{lb}$ is the initial and β_{lb} is the modified belief degree of the l^{th} consequent, then

$$\beta_{lb}=\mu_b\,\bar{\beta}_{lb},u_b=\sum_{i=1}^{|B^b|}(\phi(i,b)\sum_{j=1}^{|B_i|}\alpha_{ij})\Big/\sum_{i=1}^{|B^b|}(\phi(i,b))$$
$$\phi(i,b)=\begin{cases}1\ if\ B_i\ \text{is used in}\ R^b\\ 0\ otherwise\end{cases},\mu_b\in[0,1] \quad . \quad (6)$$
$$l=1,2,...,L$$

With the modified belief degree distribution of consequent results, a methodology is proposed to assess the utilities of feasible states through aggregating the activated rules [19]. First, it is necessary to construct the basic probability masses for all consequents.

Definition 10: Assume that $m_{l,b}$ is the basic probability mass of the l^{th} consequent in the b^{th} rule, and the unassigned segment $m_{R,b}$ consists of two parts: the $\bar{m}_{R,b}$ caused by the relative importance and the $\tilde{m}_{R,b}$ is due to the incompleteness of input data, then

$$m_{l,b}=w_b\beta_{lb},\ m_{R,b}=1-w_b\sum_{l=1}^{L}\beta_{lb}$$
$$\bar{m}_{R,b}=1-w_b,\tilde{m}_{R,b}=w_b(1-\sum_{l=1}^{L}\beta_{lb}) \quad . \quad (7)$$

Second, the aggregating approach is defined as following for multiple activated rules [17].

Definition 11: Assume that there are S activated rules. β_l indicates the belief degree of the l^{th} consequent and β_R is the unassigned segment. Moreover, the $m_{l,E(i)}$, $\bar{m}_{l,E(i)}$ and $\tilde{m}_{l,E(i)}$ represent different kinds of probability masses combining i belief rules. Then, the aggregating method is defined in (8).

$$m_{l,E(1)}=m_{l,1},\ \bar{m}_{l,E(1)}=\bar{m}_{l,1},\tilde{m}_{l,E(1)}=\tilde{m}_{l,1}$$
$$m_{l,E(i+1)}=K_{E(i+1)}(m_{l,E(i)}m_{l,i}+m_{l,E(i)}m_{R,i+1}+m_{R,E(i)}m_{l,i+1})$$
$$\bar{m}_{R,E(i+1)}=K_{E(i+1)}(\bar{m}_{R,E(i)}\,\bar{m}_{R,i+1})$$
$$\tilde{m}_{R,E(i+1)}=K_{E(i+1)}(\tilde{m}_{R,E(i)}\,\tilde{m}_{R,i+1}+\tilde{m}_{R,E(i)}\,\bar{m}_{R,i+1}+\bar{m}_{R,E(i)}\,\tilde{m}_{R,i+1})$$
$$K_{E(i+1)}=(1-\sum_{l=1}^{L}\sum_{t=1,t\neq l}^{L}m_{l,E(i)}m_{t,i+1})^{-1}$$
$$\beta_l=m_{l,E(S)}\Big/1-\bar{m}_{R,E(i+1)},\beta_R=\tilde{m}_{l,E(S)}\Big/1-\bar{m}_{R,E(i+1)} \quad . \quad (8)$$

C. Belief Preference Generation

Using the utility assessment results of feasible states, a novel approach is available to generate the information of relative preferences. Due to the uncertainties and incompleteness of state utilities, the preferences are represented in belief structure.

Definition 12: For $i\in N$ and $s\in S$, the utility of state s for DM i is represent as $U_s^i=\{(H_1^i,\beta_{1s}^i),(H_2^i,\beta_{2s}^i),...,(H_L^i,\beta_{Ls}^i),(Unknown,\beta_{Rs}^i)\}$.

Definition 13: For each $i\in N$, the preference is indicated using $P_i=\{(\succ_i,\beta_{i1}^P),(\sim_i,\beta_{i2}^P),(Unknow,\beta_{iR}^P)\}$.

Considering the monotonicity noticed in *Definition 4*, one can identify the relative preferences through comparing the belief degrees in different evaluation grades for utilities of each two feasible states.

Definition 14: For $i\in N$ and $s,t\in S$, the DM i 's preference form s to t is indicated as $P_i(s,t)$, where

$$
\begin{aligned}
&P_i(s,t) = \{(\succ_i, \beta_{i1}^{s,t}), (\sim_i, \beta_{i2}^{s,t}), (Unknow, \beta_{iR}^{s,t})\} \\
&\beta_{i1}^{s,t} = \sum_{a=1}^{L}\sum_{b=1}^{L}(\beta_{as}^{i}\beta_{bt}^{i}\varphi_1(a,b)),\ \varphi_1(a,b) = \begin{cases} 1, & if\ \ a<b \\ 0, & otherwise \end{cases} \\
&\beta_{i2}^{s,t} = \sum_{a=1}^{L}\sum_{b=1}^{L}(\beta_{as}^{i}\beta_{bt}^{i}\varphi_2(a,b)),\ \varphi_2(a,b) = \begin{cases} 1, & if\ \ a=b \\ 0, & otherwise \end{cases} \\
&\beta_{iR}^{s,t} = \sum_{b=1}^{L}\beta_{Rs}^{i}\beta_{bt}^{i} + \sum_{a=1}^{L}\beta_{Rt}^{i}\beta_{as}^{i} + \beta_{Rs}^{i}\beta_{Rt}^{i} \\
&\beta_{i1}^{s,t} \in [0,1], (\beta_{i1}^{s,t} + \beta_{i1}^{t,s}) \in [0,1]
\end{aligned} \quad . (9)
$$

IV. Conflict Analysis Methodolgy

The new preference structure provides an efficient tool to measure the motivations of each DM to make a state move with subjective, uncertain and incomplete decision support information. Accordingly, the UIs of the conflict model with belief preference are different from the traditional deterministic one, as defined as follows.

Definition 15: Assume that γ_i is the belief threshold value of the DM i, for $i \in N$ and $s \in S$, the DM i 's reachable list of UIs in belief preference structure from state s is represented as $R_i^{+\gamma_i}(s) = \{t \in S \mid (s,t) \in A_i \wedge \beta_{i1}^{t,s} \geq \gamma_i\}$.

According to the definition of UIs in belief preference structure, new stability definitions, including Nash (R), general metarational (GMR), symmetric metarational (SMR) and sequentially (SEQ), are proposed based on their standard conceptions [10].

Definition 16: For $i \in N$, a state $s \in S$ is R stable for DM i iff $R_i^{+\gamma_i}(s) = \varnothing$.

Definition 17: In a conflict model with 2 DMs, for $i, j \in N$, the other stability definitions are:

(1) a state $s \in S$ is GMR stable for DM i iff for $\forall s_1 \in R_i^{+\gamma_i}(s)$, $\exists s_2 \in R_j(s_1) \wedge s \in R_i^{+\gamma_i}(s_2)$;

(2) a state $s \in S$ is SMR stable for DM i iff for $\forall s_1 \in R_i^{+\gamma_i}(s)$, $\exists s_2 \in R_j(s_1) \wedge s \in R_i^{+\gamma_i}(s_2)$, and for $\forall s_3 \in R_i(s_2)$, $s \in R_i^{+\gamma_i}(s_3)$; and

(3) a state $s \in S$ is SEQ stable for DM i iff for $\forall s_1 \in R_i^{+\gamma_i}(s)$, $\exists s_2 \in R_j^{+\gamma_j}(s_1) \wedge s \in R_i^{+\gamma_i}(s_2)$.

Using the proposed new definitions of stabilities, it is available to compute different types of equilibriums, which suggest the potential compromises for all DMs, in a specified interactive decision issue. Further, an illustrative example for system portfolio selection is investigated in next section to demonstrate this methodology.

V. Illustrative Example

An interactive decision case to design the architecture for a WSoS of the national maritime threat response (MTR) is utilized as the illustrative example to demonstrate the validity of this proposed methodology. In order to counter the threats that potential terrorists employ a small boat attack (SBA) against high value units, a successful MTR is required to possess three validated capabilities, including: 1) the command, control, computers, communication, intelligence, surveillance and reconnaissance (C4ISR) capability; 2) the prepare the battle space (PBS) capability; and 3) the engage (ENG) capability [2] [23]. Further, available system alternatives, depicted as options in this decision issue, are related to each required capability. For the C4ISR, the related alternatives include the area commander (AC), local commander (LC), objective-oriented command structure (OOC) and problem-solving command structure (PSC). Additionally, PBS focuses on small escorts with surface search radar (E/R) and the escort teams employing a visual detection scheme (T/V) while ENG involves the weapons organic to PBS (OW), armed helicopters (AH) and unarmed unmanned surface vehicle (USV).

The objectives for the system portfolio selection include maximizing performance, minimizing cost and improving cost effectiveness. According to the model framework mentioned above, the performance, measured by probability of success, and cost effectiveness are abstracted as DM1 and DM2, while the cost is utilized as the resource constraint to filter reasonable system portfolios which are represented as feasible states. In order to simplify the GMCR, the part related to ENG of the selected portfolio is fixed as OW, AH and USV. The alternatives for PBS are indicated as options of DM1 and the remainders are allocated to DM2. With the resource constraint illustrated in [23] and the state transitions proposed in [2], the graph model, of which the node information is listed in Table I and arc information is demonstrated in Fig. 1, is available.

TABLE I.
Feasible System Portfolios for the MTR Decision Making Issue

DM	Option	1	2	3	4	5	6	7	8
1	E/R	Y	Y	Y	Y	N	N	N	N
	T/V	N	N	N	N	Y	Y	Y	Y
2	AC	Y	N	Y	N	Y	N	Y	N
	LC	N	Y	N	Y	N	Y	N	Y
	PSC	Y	Y	N	N	Y	Y	N	N
	OOC	N	N	Y	Y	N	N	Y	Y

In addition, the evaluation grades of utility assessment results are divided discretely to construct preference belief structure. For each DM, the assessment sets are both defined as a three-tuple structure $< A, B, C >$ in this case.

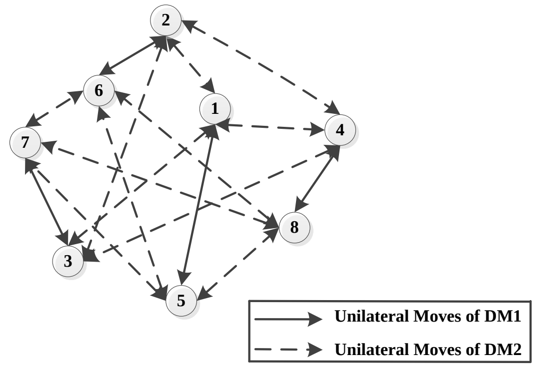

Figure 1. GMCR for the MTR decision making issue.

According to the experimental results in [23], the performance of all feasible states are available in both the probability of success P_s and cost effectiveness z. Using the statistical results and expertise, the belief rule base for the utility of i^{th} DM DU_i, in which the weights of all rules are identical, is constructed, as displayed in Table II.

TABLE II
BELIEF RULE BASE FOR THE MTR DECISION MAKING ISSUE

No.	Antecedents	Consequents
1	$P_s = 1$	$DU_1 = \{(A,1),(B,0),(C,0)\}$
2	$P_s = 0.75$	$DU_1 = \{(A,0.75),(B,0.25),(C,0)\}$
3	$P_s = 0.46$	$DU_1 = \{(A,0.25),(B,0.5),(C,0.25)\}$
4	$P_s = 0.24$	$DU_1 = \{(A,0),(B,0),(C,1)\}$
5	$z = 86.4$	$DU_2 = \{(A,0.95),(B,0),(C,0)\}$
6	$z = 75.23$	$DU_2 = \{(A,0.75),(B,0.25),(C,0)\}$
7	$z = 38.1$	$DU_2 = \{(A,0.25),(B,0.5),(C,0.25)\}$
8	$z = 35.1$	$DU_2 = \{(A,0),(B,0),(C,1)\}$

Based on the support information in the related literature [23], definitions in Section III-B and the belief rules above, the utilities of all feasible states for each DM, depicted as belief degrees in different evaluation grades, can be computed. The results are shown in Table III.

TABLE III
UTILITIES OF FEASIBLE STATES FOR DMS

No.	P_s	z	DU_1	DU_2
1	0.76	44.7	0.7580, 0.2420, 0	0.2923, 0.4833, 0.2244
2	0.4	61.2	0.2025, 0.4051, 0.3924	0.6134, 0.3274, 0.0592
3	0.63	79.9	0.5860, 0.3416, 0.0724	0.8729, 0.1507, 0
4	0.63	35.9	0.5860, 0.3416, 0.0724	0.0270, 0.0541, 0.9189
5	0.81	49.8	0.8130, 0.1870, 0	0.3652, 0.4504, 0.1844
6	0.24	48.1	0, 0, 1	0.3366, 0.4636, 0.1998
7	0.71	86.4	0.7312, 0.2629, 0.0059	0.95, 0, 0
8	0.63	35.1	0.5860, 0.3416, 0.0724	0, 0, 1

Using the utility assessment results, one can generate the belief preference for each DM with the definitions proposed in Section III-C. Assume that $\gamma_1 = \gamma_2 = 0.3$, the new graph model consists of feasible states and UIs are available, as demonstrated in Fig. 2.

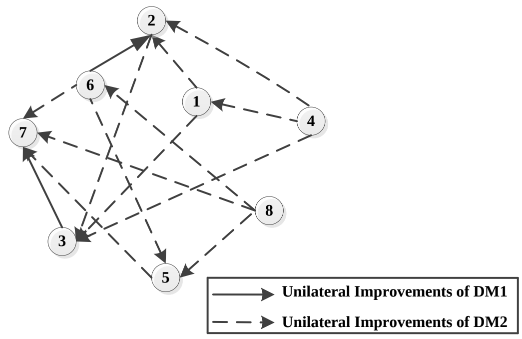

Figure 2. New graph model consists of feasible states and UIs.

With the two graph models above, it is not difficult to compute the conflict analysis results. According to the *Definitions 16-17*, one can identify all kinds of stabilities and equilibriums (E), as illustrated in Table IV.

TABLE IV
STABILITY ANALYSIS RESULTS FOR GMCR

State	R			GMR			SMR			SEQ		
	1	2	E	1	2	E	1	2	E	1	2	E
1	√			√			√			√		
2	√			√			√			√		
3		√		√	√	√	√	√	√		√	
4	√			√			√			√		
5	√			√			√			√		
6												
7	√	√	√	√	√	√	√	√	√	√	√	√
8	√			√			√			√		

VI. CONCLUSIONS

This paper provides a novel methodology for interactive decision making problems. With the GMCR framework, an evidence-integrating approach based on RIMER is utilized to assess utilities of feasible states for DMs, and the belief preferences are available using the assessment results. In order to deal with the belief preferences, the authors present new definitions of the UI and four kinds of stabilities. Finally, the MTR decision making case is studied as an illustrative example to demonstrate the validity of this proposed methodology.

According to the conflict analysis results of this mentioned issue, one can identify that the probable equilibriums, including the states 3 and 7, possess excellent performances in both probability of success and cost effectiveness. It is demonstrated that the proposed methodology can be utilized to find out satisfactory system portfolios in an interactive decision making procedure.

Although this research has provided a validated solution to handle the subjectivity, epistemic uncertainty and incompleteness in GMCR, a lot of problems still need to be worked on for a better decision. The next study of the authors aims at expanding the proposed methodology to multiple DMs and investigating the influences to analysis results due to the changing of the belief rule base and threshold parameters.

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